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DYNAMIC STRAIN OF A CONDUCTING HALF SPACE WITH A CAVITY IN A STRONG MAGNETIC FIELD

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The mechanical excitation of dia(para)magnetics in a static magnetic field creates an induced (rotational) current inside the body, which leads to the formation of Lorentz body forces, which are calculated by a tensor of Maxwellian stresses, which introduce large corrections in the stress state of the body.

Below we examine a conducting elastic half space with tunnel cavities which is subjected to mechanical excitation in a homogeneous static magnetic field. The corresponding magnetoelastic problem is reduced to a singular integral equation, which is solved numerically with the use of the method of mechanical quadratures. Calculated results are presented, which characterize the stress concentrations at the contour of the cavity as a function of the configuration of the aperture, the magnitude of the applied magnetic field, and the frequency of the excitation.

1. Basic Linear Magnetoelastic Equations and Formulation of the Problem. The total system of magnetoelastic equations include [1-3] the equations of motion

$$\partial_j\sigma_{ij} + \rho_e E_i + (\mathbf{j} \times \mathbf{B})_i = \rho\partial^2 u_i/\partial t^2 \quad (i, j = 1, 2, 3); \tag{1.1}$$

Maxwell's equations

$$\mathrm{rot}\,\mathbf{E} + \partial\mathbf{B}/\partial t = 0,\ \mathrm{rot}\,\mathbf{H} - \partial\mathbf{D}/\partial t = \mathbf{j},\ \mathrm{div}\,\mathbf{D} = \rho_e,\ \mathrm{div}\,\mathbf{B} = 0 \tag{1.2}$$

and the material equations

$$\begin{gathered}\mathbf{D} = \varepsilon\mathbf{E} + \alpha(\mathbf{v} \times \mathbf{H}),\ \mathbf{B} = \mu_e\mathbf{H} - \alpha(\mathbf{v} \times \mathbf{E}),\\ \mathbf{j} = \rho_e\mathbf{v} + \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}),\ \alpha = \varepsilon\mu_e - \varepsilon_0\mu_0,\\ \sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk},\ \varepsilon_{ij} = (1/2)(\partial_j u_i + \partial_i u_j),\\ \partial_i = \partial/\partial x_i,\ \mathbf{v} = \partial\mathbf{u}/\partial t \quad (i,\ j,\ k = 1, 2, 3).\end{gathered} \tag{1.3}$$

The boundary conditions on the separation surface between two media have the form

$$\begin{gathered}[\mathbf{E} + \mathbf{v} \times \mathbf{B}]_\tau = 0,\ [\mathbf{H} - \mathbf{v} \times \mathbf{D}]_\tau = 0,\\ [\mathbf{B}]_n = 0,\ [\mathbf{D}]_n = 0,\ [\sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \rho_e\mathbf{v}]_n = 0,\\ [\sigma_{ij} + t_{ij}]n_j = X_{in} \quad (i, j, k = 1, 2, 3),\\ t_{ij} = E_iD_j + H_iB_j - (1/2)\delta_{ij}(E_kD_k + B_kH_k).\end{gathered} \tag{1.4}$$

Here $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{H}$, $\mathbf{B}$ are the intensities and inductions, correspondingly of the electric and magnetic fields; ε, ε_0 and μ_e, μ_0 are the electric and magnetic permeabilities in the material and in a vacuum; ρ_e is the spatial density of the electric charge; $\mathbf{j}$ is the current density; ρ is the density of the material; u_i and σ_{ij} are the mechanical displacements and stresses; the X_{in} are the components of the external surface load; μ and λ are the Lamé constants; δ_{ij} is the Kronecker delta; and the symbol [ ] is a jump in the corresponding quantity at the separation line of the media.

Let a static magnetic field $\mathbf{H}^0$ act on a quiescent magnetoelastic medium. The external excitation creates a body strain and the creation of an electromagnetic field which can be

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described by small fluctuations $\mathbf{e} = (e_1, e_2, e_3)$ and $\mathbf{h} = (h_1, h_2, h_3)$. Hereafter we will assume that the field is quasistatic ($\mathbf{D} = 0$ and $\partial\mathbf{D}/\partial t = 0$).

If we assume $\mathbf{H} = \mathbf{H}^0 + \mathbf{h}$ and $\mathbf{E} = \mathbf{e}$, the system (1.1)-(1.4) can be written as follows

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \mu_e(\mathbf{j}\times\mathbf{H}^0) = \rho\partial^2\mathbf{u}/\partial t^2,$$
$$\mathrm{rot}\,\mathbf{h} = \mathbf{j},\ \mathrm{rot}\,\mathbf{e} = -\mu_e\partial\mathbf{h}/\partial t,\ \mathrm{div}\,\mathbf{h} = 0,$$

$$\mathbf{b} = \mu_e\mathbf{h},\ \mathbf{j} = \sigma(\mathbf{e} + \mu_e\mathbf{v}\times\mathbf{H}^0),$$
$$[\mathbf{e} + \mu_e\mathbf{v}\times\mathbf{H}^0]_\tau = 0,\ [\mathbf{h}]_\tau = 0,\ [\sigma(\mathbf{e} + \mu_e\mathbf{v}\times\mathbf{H}^0)]_n = 0,$$
$$[\mu_e\mathbf{h}]_n = 0,$$
$$\left[\sigma_{ij} + \mu_e\left(H_i^0h_j + H_j^0h_i - \delta_{ij}H_k^0h_k\right)\right]n_j = X_{in}.$$

For some materials (Al and Cu, for example), it is convenient to simplify the model and assign ideal conductivity ($\sigma \to \infty$). In this case we arrive at a closed system of equations

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \mu_e\,\mathrm{rot}\,\mathbf{h}\times\mathbf{H}^0 = \rho\partial^2\mathbf{u}/\partial t^2, \tag{1.5}$$
$$\mathbf{h} = \mathrm{rot}\,(\mathbf{u}\times\mathbf{H}^0),\ \mathbf{e} = -\mu_e(\mathbf{v}\times\mathbf{H}^0),\ [\mathbf{h}]_\tau = 0,$$
$$[\mu_e\mathbf{h}]_n = 0,$$
$$\left[\sigma_{ij} + \mu_e\left(H_i^0h_j + H_j^0h_i - \delta_{ij}H_k^0h_k\right)\right]n_j = X_{in}.$$

We will assume that the magnetoelastic medium is inhomogeneous, namely: the medium contains the cylindrical cavities L_j ($j = 1,\ldots,k$) along the x_3 axis, and the vector of the initial magnetic field $\mathbf{H}^0 = (0;\ H_0;\ 0)$ (Fig. 1). The corresponding static field is described by the system of equations

$$\mu\nabla^2u_i + (\lambda+\mu)\,\partial_i\theta = 0,\ \ \theta = \partial_1u_1 + \partial_2u_2,\ \ \mu\nabla^2u_3 = 0,\ \ \nabla^2 = \partial_1^2 + \partial_2^2\ \ (i=1,\ 2)$$

and the boundary conditions

$$\sigma_{11}\cos\psi + \sigma_{12}\sin\psi = X_{1n} + \frac{\varkappa\mu_0}{2}H_0^2(1 + \varkappa\sin^2\psi)\cos\psi,$$
$$\sigma_{21}\cos\psi + \sigma_{22}\sin\psi = X_{2n} + \frac{\varkappa\mu_0}{2}H_0^2(1 + \varkappa\sin^2\psi)\sin\psi,$$
$$\sigma_{31}\cos\psi + \sigma_{32}\sin\psi = X_{3n},\ \varkappa = \mu_e/\mu_0 - 1,$$
$$H_1^* = (\varkappa/2)H_0\sin2\psi,\ \ H_2^* = H_0(1 + \varkappa\sin^2\psi).$$

Here ψ is the angle between the normal to the contour L_j and the axis Ox_1 (Fig. 1) the asterisk refers to the inside of the cavity.

Thus, the static field is determined by the usual static equations of the theory of elasticity, where the two-dimensional problem is divided into states of plane and antiplane strain. For materials such as copper, aluminum, and several others, the magnetic permeability is almost the same as the magnetic permeability of a vacuum. Therefore we can set $\kappa = 0$. In this case the previous static magnetic field generally does not effect the stress state of the body.

We now determine the fluctuation field. From (1.5) we obtain the following equations: for the plane strain, the equations of motion are

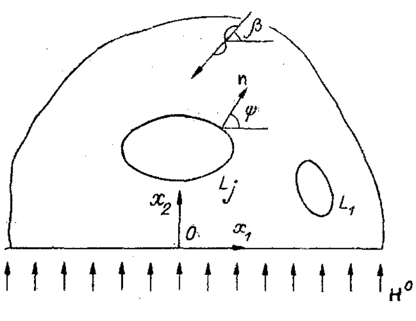

Fig. 1

$$(1+\chi^2)\nabla^2 u_1 + \sigma_* \partial_1 \theta = \frac{1}{c_2^2}\frac{\partial^2 u_1}{\partial t^2}, \quad c_2 = \sqrt{\frac{\mu}{\rho}},$$
$$\nabla^2 u_2 + \sigma_* \partial_2 \theta = \frac{1}{c_2^2}\frac{\partial^2 u_2}{\partial t^2}, \quad \chi^2 = \frac{\mu_e H_0^2}{\mu}, \quad \sigma_* = \frac{\lambda+\mu}{\mu},$$

the components of the electromagnetic field are

$$h_1 = H_0 \partial_2 u_1,\ h_2 = -H_0 \partial_1 u_1,\ h_3 = 0,\ e_1 = e_2 = 0,$$
$$e_3 = -\mu_e H_0 \partial u_1/\partial t,$$

and the boundary conditions on L_j $(j = 1,\ldots,k)$ are

$$\sigma_{11}\cos\psi + \sigma_{12}\sin\psi = X_{1n} + \mu_0 \varkappa H_0 (h_2 + $$
$$+ \varkappa \sin\psi (h_1 \cos\psi + h_2 \sin\psi))\cos\psi,$$
$$\sigma_{21}\cos\psi + \sigma_{22}\sin\psi = X_{2n} + \mu_0 \varkappa H_0 (h_1 \cos\psi + $$
$$+ h_2 \sin\psi)(1 + \varkappa \sin^2\psi),$$
$$h_1^* = h_1(1+\varkappa\cos^2\psi) + (\varkappa/2) h_2 \sin 2\psi, \quad h_2^* = h_2(1+\varkappa\sin^2\psi) + (\varkappa/2) h_1 \sin 2\psi;$$

for the antiplane strain, the equations of motion are

$$\nabla^2 u_3 + \chi^2 \partial_2^2 u_3 = \frac{1}{c_2^2}\frac{\partial^2 u_3}{\partial t^2}, \tag{1.6}$$

the components of the magnetic field are

$$h_1 = h_2 = 0,\ h_3 = H_0 \partial_2 u_3,\ e_1 = -\mu_e H_0 \partial u_3/\partial t,\ e_2 = e_3 = 0, \tag{1.7}$$

and the boundary conditions on L_j $(j = 1,\ldots,k)$ are

$$\sigma_{31}\cos\psi + \sigma_{32}\sin\psi = X_{3n}, \quad h_3^* = h_3. \tag{1.8}$$

Below we will examine the problem of the antiplane strain (1.6)-(1.8) for a conducting half space $x_2 \geq 0$ with cylindrical cavities L_j along x_3 (see Fig. 1). Let the half space be free of forces and bounded by a vacuum, where the static magnetic field in the vacuum is $(0;\ H_0^*;\ 0)$, but in the medium the field is $(0;\ H_0;\ 0)$, where $H_0 = \mu_0 H_0^*/\mu_e$. For a mechanical excitation, we take either the shear load $X_{3n} = \mathrm{Re}(X_3 e^{-i\omega t})$, which is independent of the coordinate x_3 and which acts on the cavity surfaces or else we take a shear displacement wave coming in from infinity:

$$u_3^0 = \mathrm{Re}\left(U_3^0 e^{-i\omega t}\right), \quad U_3^0 = U\exp\left(-i\gamma\left(x_1\cos\beta + x_2\sin\beta\right)\right), \tag{1.9}$$
$$U = \mathrm{const}, \quad \gamma_2 = \omega/c_2, \quad \gamma = \gamma_2/\left(\sqrt{1+\chi^2\sin^2\beta}\right).$$

The mechanical field in the half space with the cavities is composed of the field of an incoming wave (1.9), the field of a reflected wave

$$u_3^1 = \mathrm{Re}\left(U_3^1 e^{-i\omega t}\right), \quad U_3^1 = U\exp\left(-i\gamma\left(x_1\cos\beta - x_2\sin\beta\right)\right) \tag{1.10}$$

and a scattered field, which is represented in the form

$$u_3 = \mathrm{Re}\,(U_3 e^{-i\omega t}), \tag{1.11}$$
$$U_3 = \int_L p(\zeta)\left(H_0^{(1)}(\gamma_2 r_1) + H_0^{(1)}(\gamma_2 r_1^*)\right) ds,$$
$$\zeta_1 = \xi_1 + \frac{i\xi_2}{\sqrt{1+\chi^2}}, \quad z_1 = x_1 + \frac{ix_2}{\sqrt{1+\chi^2}},$$
$$r_1 = |\zeta_1 - z_1|, \quad r_1^* = |\bar{\zeta}_1 - z_1|, \quad \zeta = \xi_1 + i\xi_2 \in L = UL_j.$$

Here $p(\zeta) = \{p_j(\zeta),\ \zeta \in L_j\}$ is the unknown density; $H_n^{(1)}(x)$ is the n-th order Hankel function of the first kind; ds is an element of arc of the contour L; and integration is counterclockwise. The representation (1.11) automatically satisfies the condition $\sigma_{32} = 0$ at the boundary of the half space and the radiation condition, and the function u_3 is the solution of Eq (1.6).

2. Integral Equation of the Boundary Problem. The boundary condition (1.8) on L_j is represented in the amplitudes

$$\left(S_{13} + S_{13}^0 + S_{13}^1\right)\cos\psi + \left(S_{23} + S_{23}^0 + S_{23}^1\right)\sin\psi = X_3 \tag{2.1}$$

where S_{i3}, $S^0{}_{i3}$, and $S^1{}_{i3}$ are the corresponding amplitudes of the quantities σ_{i3}, $\sigma^0{}_{i3}$, and $\sigma^1{}_{i3}$.

By calculating the stresses from Eqs. (1.3) with a consideration of (1.11) and substituting their limiting values for $z \to \zeta_0 \in L_j$ into the boundary condition (2.1), we come to the integral equation for $p(\zeta)$:

$$p(\zeta_0) + \int_L p(\zeta) G(\zeta; \zeta_0)\, ds = N(\zeta_0), \tag{2.2}$$

$$G(\zeta; \zeta_0) = \frac{2}{i\pi\eta(\psi_0)} \operatorname{Re}\left(\frac{c(\psi_0)}{\zeta_1 - \zeta_{10}}\right) + \frac{\gamma_2}{\eta(\psi_0)} \left(H_1(\gamma_2 r_{10}) \operatorname{Re}\left(c(\psi_0) e^{-i\alpha_{10}}\right) + \right.$$

$$\left. + H_1^{(1)}(\gamma_2 r_{10}^*) \operatorname{Re}\left(c(\psi_0) e^{-i\alpha_{10}^*}\right)\right),$$

$$N(\zeta) = \frac{1}{\mu\eta(\psi)} X_3 + \frac{i\gamma}{\eta(\psi)} \left(\cos(\beta - \psi) U_3^0 + \cos(\beta + \psi) U_3^1\right),$$

$$\eta(\psi) = -2i \operatorname{Im}\left(\frac{c(\psi)}{a(\psi)}\right), \quad c(\psi) = \cos\psi + \frac{i \sin\psi}{\sqrt{1 + \chi^2}},$$

$$a(\psi) = \frac{dc(\psi)}{d\psi}, \quad r_{10} = |\zeta_1 - \zeta_{10}|, \; r_{10}^* = |\bar{\zeta}_1 - \zeta_{10}|,$$

$$\alpha_{10} = \arg(\zeta_1 - \zeta_{10}), \quad \alpha_{10}^* = \arg(\bar{\zeta}_1 - \zeta_{10}).$$

We note that in the absence of a previous magnetic field ($\chi = 0$), Eq. (2.2) is a Fredholm integral equation of the second kind. If, however, $\chi > 0$, then we obtain a singular integral equation of the second kind.

3. Stress Concentration. In determining the wave field of stresses in the body, it is necessary to consider the Maxwellian contributions. Therefore, in the general case, the shear stresses in a plane perpendicular to the cavity contour are determined by

$$\tau_s = -\left(\sigma_{13} + \sigma_{13}^0 + \sigma_{13}^1 + t_{13}\right) \sin\psi + \left(\sigma_{23} + \sigma_{23}^0 + \sigma_{23}^1 + t_{23}\right) \cos\psi, \tag{3.1}$$

$$\tau_s = \operatorname{Re}\left(T e^{-i\omega t}\right).$$

The Maxwellian stresses t_{i3} in our case have the form

$$t_{13} = 0, \quad t_{23} = \mu_e H_0^2 \partial_2 u_3. \tag{3.2}$$

By introducing Eqs. (1.3), (1.9)-(1.11), and (3.2) into Eq. (3.1), we determine the amplitude of the stresses at the point $\zeta_0 \in L$:

$$T(\zeta_0) = -2i\mu\sqrt{1 + \chi^2}\, p(\zeta_0) \operatorname{Re}\left(\frac{c(\psi_0)}{a(\psi_0)}\right) + \int_L p(\zeta) K(\zeta; \zeta_0)\, ds + \tag{3.3}$$

$$+ i\mu\gamma\left(\sin(\psi_0 - \beta) U_3^0 + \sin(\psi_0 + \beta) U_3^1\right),$$

$$K(\zeta; \zeta_0) = \frac{2i\mu\sqrt{1 + \chi^2}}{\pi} \operatorname{Im}\left(\frac{c(\psi_0)}{\zeta_1 - \zeta_{10}}\right) - \mu\gamma_2\left(H_1(\gamma_2 r_{10})\, b + H_1^{(1)}(\gamma_2 r_{10}^*)\, b^*\right),$$

$$b = \cos\alpha_{10} \sin\psi_0 - \sqrt{1 + \chi^2} \cos\psi_0 \sin\alpha_{10},$$

$$b^* = \cos\alpha_{10}^* \sin\psi_0 - \sqrt{1 + \chi^2} \cos\psi_0 \sin\alpha_{10}^*.$$

Example. Let the half space be weakened by a cavity with an elliptical cross section $\xi_1 = a_1 \cos\phi$, and $\xi_2 = h + b_1 \sin\phi$. We will assume that the surface of the cavity is free from stresses, and that a magnetoelastic shear wave (1.9) comes from infinity along the x_2 axis. In order to determine the stresses at the contour of the cavity from Eq. (3.3), it is necessary to know the function $p(\zeta)$ which is determined from the integral equation (2.2). The latter is solved numerically by the method of mechanical quadratures [4]. The results calculating the quantities $\langle T \rangle = T/T_0$ as a function of $\gamma_2 R$, where $T_0 = \mu U\gamma$ and $R = (a_1 + b_1)/2$, are shown in Figs. 2 and 3 for the following parameters: $a_1 = 1$, $b_1 = 0.75$, and $h = 1.75$. Curves in Fig. 2 correspond to the point of the contour $\phi = \pi/2$; curves in Fig. 3 correspond to the point $\phi = 0$. Curves 1-3 are constructed for $\chi = 0$, 0.5, and 1. Curve 4 corresponds to a circular cavity $a_1 = b_1 = 1$, $h = 1.5$, $\chi = 0$, $\beta = 7\pi/8$, and $\phi = -\pi/2$. The points show corresponding data obtained by a completely different method [5].

The results of the calculations show that not considering the previous magnetic field in analyzing the dynamic strain of a body can lead to a calculated strength of the body that is larger than the actual strength.

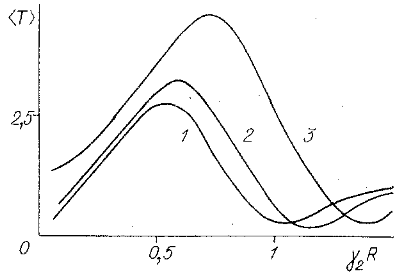

Fig. 2

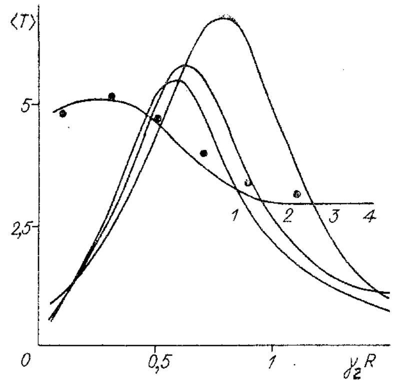

Fig. 3

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TRANSMISSION OF A PRESSURE PULSE TO METALLIC AND DIELECTRIC TARGETS IRRADIATED BY A NEODYMIUM LASER IN THE FREE GENERATION REGIME

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Pressure oscillations which are measured by a piezotransducer on the back side of targets are observed when concentrated energy fluxes, e.g., from laser radiation, interact with a material [1]. Zhiryakov et al. [1] observed pressure oscillations on a lead target for $J \simeq 2$ MW/cm^2, which disappeared rapidly when J was increased. Possible mechanisms for creating the oscillations due to an autovibration regime of self-screening or spurts of absorption in the plasma of light-eroding flares in the unstable evaporation regime have been analyzed in detail [2, 3].

Experiments were conducted in the VIKA vacuum chamber for a detailed investigation of pressure oscillations in targets [4]. Pulsed laser radiation with a wavelength of 1.06 μm and a pulse width at half maximum of $3 \cdot 10^{-4}$ sec acted on metallic and dielectric targets in a chamber whose pressure could be varied from 10^5 to 10^{-2} Pa. The diameter of the radiation spot in most cases was 6 mm; the target diameter was 20 mm.

The target 1 was mounted on a piezotransducer (Fig. 1), which operated as a voltage source. Its main feature was a long wave guide behind the piezoelement 2, which gave a read time up to 1.5 msec for both the initial and the reflected signal. The transducer was well shielded from electric and acoustic fields. A wideband amplifier 6, designed with an RC input on the order of 1-100 sec, was connected directly to the transducer in the vacuum chamber, which provided a very small charge loss from the piezoelement during the read time.

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